On Trellis Structure of Error Correction Coding

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Abstract

The modern trends in communication field points towards achieving the fastest way of data transmission through communication line, reaching Shannon's Channel Limit. The key to these developments is error correction coding technique which is currently employed in broadband satellite communication and data storage. Constructing Trellis for the codes - a graphical way of code analysis that allows us to avoid repeating the same computations over and again - reduces the decoding complexity, thereby improves transmission efficiency. The paper will investigate the way trellises are constructed for different types of codes, how their complexity can be reduced and how they are used to correct errors on transmission channels. This includes study of trellis pattern for different coders – for both convolutional and block codes and its implementation in a practical communication channel using Monte Carlo Simulation technique. A brief comparison of bit error rate for hard and soft decision decoding techniques for selected coders and for different message lengths is performed with BER Vs SNR plots.

Keywords

Error Correction codes, Trellis representation, Convolutional coding, Hard decision Viterbi decoding, Soft Decision Viterbi Decoding, Monte Carlo Simulation

1 Introduction

In communication system, coding theory dealt with the design and evaluation of efficient signalling schemes for reliable data transmission and storage. It is applied in telephone-line modems, where increasing transmission speeds introduce high levels of noise; compact-disk recorders, in which error is inherent in the production process; and deep-space probes, in which large lag times confound the problems associated with transmission error .(Trachtenberg,2000)

In 1948 Claude Shannon showed that it is possible to transmit information over a noisy channel with arbitrarily small probability of error, at rates up to the capacity of the channel (Shannon, 1948). Error correcting codes are used for reliable transmission of information over noisy channels, which encode input in such a way that errors can be detected and corrected at the receiving site.

The design of error correcting codes that minimize the probability of error on one hand and maximize the information rate on other hand is little complicated. Also it should be capable of reconstructing the most likely transmitted codeword from the error-corrupted sequence observed at the output of a noisy channel.

This paper approach both of these problems by the study of decoding techniques based on graphs. A code which has a low error probability and a reasonably high information rate, and can be decoded efficiently by means of a trellis, is developed. The trellis may be thought of as a constrained finite-state automaton; diagram that allows us to avoid repeating the same computations over and again (something similar to the FFT algorithms); it was originally introduced by Forney in 1967 to explain the Viterbi decoding algorithm.

A Forward Error Correction code is a redundant data added to the message at the sender side and the receiver can use the extra information to discover the location of the error and correct them. Convolutional codes are processed on a bit-by-bit basis, and only cause a processing delay corresponding to a few bit periods. It has memory shift registers. Block codes are processed on a block-by-block basis.

2 Trellis Formation

2.1 Convolutional Encoding

A convolutional code can be defined by kn polynomials and a k*n generator-polynomial matrix. For k=1, a convolutional code can be compactly defined by the generator polynomial matrix,

$G(x) = [g1(x), g_2(x),\ldots\ldots g_n(x)]$

For a simple code G = [5,7] ; R= ½ ; K=3

$5 = 101 \equiv 1+x^2 \quad = g_1(x)$

$7 = 111 \equiv 1+x+x^2 = g_2(x)$

Let $\{u\} = 0110100\ldots\ldots\ldots$

$g_1(x)\,u(x) = (1+x^2)\,(x+x^2+x^4) = x+x^2+x^4+x^3+x^4+x^6 = x+x^2+x^3+x^6$

$\{v_1\} = 0111001\ldots$

$g_2(x)\,u(x) = (1+x+x^2)\,(x+x^2+x^4) = x+x^2+x^4+x^2+x^3+x^5+x^3+x^4+x^6$

$= x+2x^2+2x^3+2x^4+x^5+x^6 = x+x^5+x^6$

$\{v_2\} = 0100011\ldots..$

Therefore, $\{v\}$ = 00 11 10 10 00 01 11 ……….

Here the information sequence cannot be identified in the code sequence. Therefore it is non-systematic. Non-systematic codes are preferred when viterbi decoding is used as they offer maximum $d_{free.}$

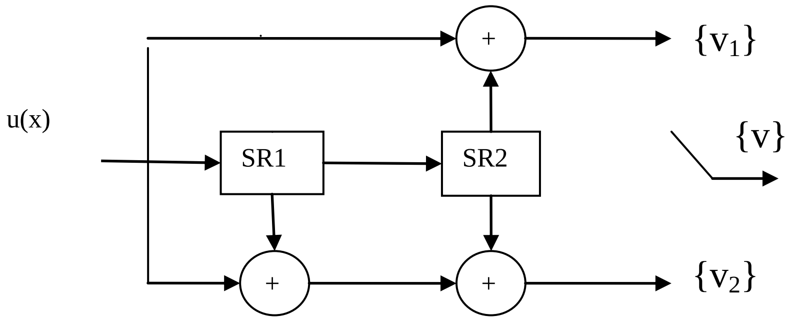

Figure 1: Convolutional Coder G=[5,7]

Original State	Input	Output	Final State
00	0	00	00
	1	11	10
01	0	11	00
	1	00	10
10	0	01	01
	1	10	11
11	0	10	01
	1	01	11

Table .1.FSM for G=[5,7]

Take input sequence as 10101101

V1 = 10000110
V2 = 11010000
V = 11 01 00 01 00 10 10 00

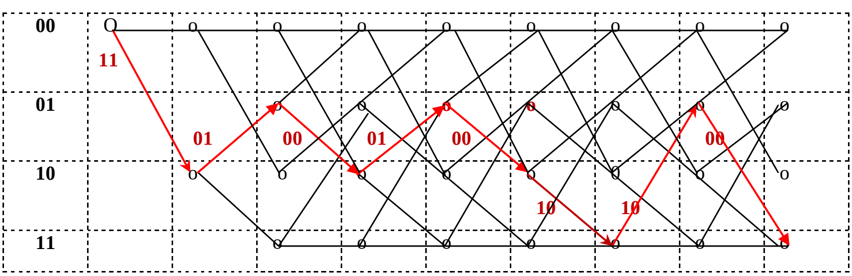

Figure 2: Trellis for convolutional encoder G=[5,7]

3 Decoding

A convolutional encoder is basically a finite-state machine; hence the optimum decoder is a Maximum-Likelihood Sequence Estimator (MLSE). Therefore, optimum decoding of a convolutional code involves a search through the trellis for the most probable sequence. Depending on whether the detector performs hard or soft decision decoding, the corresponding metric in the trellis search may be either a Hamming metric or a Euclidean metric. (Proakis,1995)

The Viterbi algorithm operates frame by frame over a finite number of frames. At any frame the decoder does not know the node the encoder reached, so it labels the possible nodes with metrics- in this case the running Hamming distance between the trellis path and the input sequence. In the next frame the decoder uses these metrics to deduce the most likely path and drop other paths (Wade, 1994).

Hard-decision Viterbi decoding seeks a trellis path which has minimum Hamming distance from a quantized channel output sequence.

Hard Decision Viterbi Decoding For a simple non recursive coder G = [5,7] ; R= ½ ; K=3

Let i = 1 0 1 0 1 1 0 1

V = 11 01 00 01 11 10 10 00

r = 11 **1**1 00 01 11 10 1**1** 00

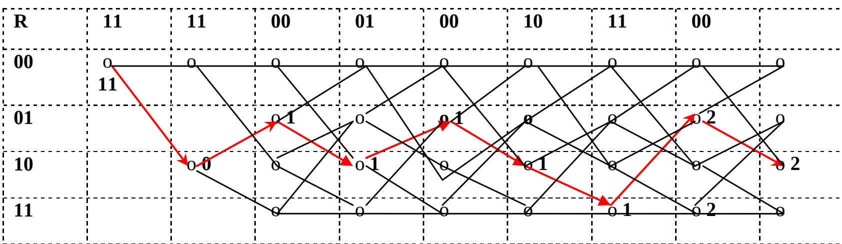

Figure 3: Trellis showing full Viterbi decoding for G=[5,7]

Thus the minimum distance unique path is retrieved and from the trellis we will get the decoded output as: 1 0 1 0 1 1 0 1

Soft Decision Viterbi Decoding For a simple non recursive coder G = [5,7] ; R= ½ ; K=3

For a soft decision input (output of demodulator quantized to more than two levels), maximum likelihood decoding is achieved by minimizing a Euclidean distance. In terms of Viterbi algorithm trellis, this amounts to computing successive branch metrics and accumulating their values in path metrics (Wade, 1994).

$$bm = \Sigma (r - v)^2$$

Let

i = 1 0 1 0

V = 1 1 0 1 0 0 0 1

r = 0.9, 1.1, -0.2, 1.3, 0.2,- 0.3, -0.1,0.9

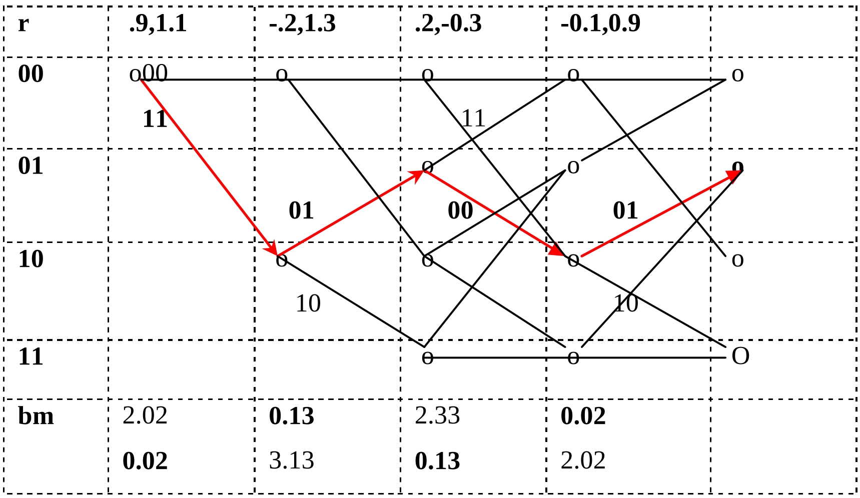

Figure 4: Trellis showing Soft Viterbi decoding for G=[5,7]

On finding the branch metrics, follow the lowest value path, discarding high difference values at each node. This will be the unique trellis path and the output values are getting decoded to original input.

3.1 Trellis representations of binary linear block codes

Trellis-based (Viterbi algorithm) decoding is one of the most efficient methods known for maximum-likelihood (ML) decoding of general binary linear block codes; Certain binary linear block codes could be represented as terminated convolutional codes, and therefore has trellis representations (Forney, 2005). Any (n, n- 1, 2) single-parity-check (SPC) code has a two-state trellis representation like shown,

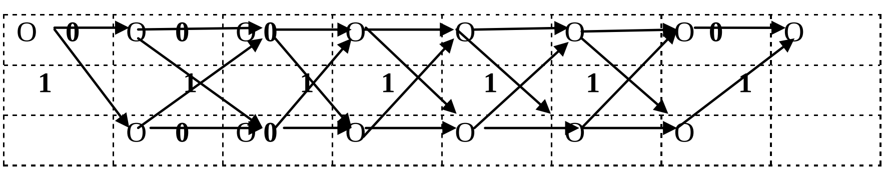

Figure 5: Trellis for Simple Block code

A generator matrix for a linear code is a binary matrix whose rows are the code words belonging to some basis for the code. A generator matrix is in Trellis Oriented Form (TOF) when for each row g $\mathcal{E}$ $\{g_1, g_2, \ldots, g_{k-1}\}$. The leading 1 (first non-zero component of a row) appears in a column before the leading 1 of any row below it. No two rows have their trailing 1(last non-zero component of the row) in the same column (University of Crete, 2009).

4 Methodology & Results

In digital communication, the matched filtering technique has been used widely to maximise the output signal by maximising the output SNR (signal to noise ratio). Here the performance of the matched filter for the Base band Binary Transmission in the presence of Gaussian noise is briefly studied and its BER (Bit Error Rate) is estimated using MATLAB simulation (Monte Carlo Simulation) for 3 different coders designed using trellis structure. The BER of the filter for different operating points is generated and plotted against the theoretical value of the BER and is compared. A comparison of the performance of coders for hard and soft decision coding techniques is discussed for different message length.

The theoretical and simulated BER gives a clear picture of the performance of system. After doing Monte Carlo simulation, from the plots it is found that the theoretical line coincides with a majority of the practically simulated points. Thus the matched filter designed is a good one and it approximates very closely to the practical results and gives high performance. The Trellis coding and Viterbi decoding are the underlying features that enables the error free transmission.

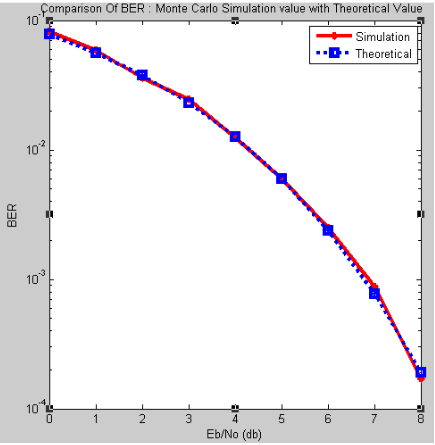

Figure 6: Monte Carlo Simulation Result comparing theoretical & Practical BER for the designed system

It is observed that for larger message length, the number of error bits is comparatively not very high, trellis found greater advantage in reducing error rate. For hard decision decoding, the coder with lower constraint length produces comparatively less error bits. For soft decision, coder with higher constraint length gives less number of error bits. In Table 2, the coder G=[133,171] gives very good performance for soft decision decoding.

Coder	Message Length	Number of error bits		BER	
		Hard Decision	Soft Decision	Hard Decision	Soft Decision
G=[5,7]	1000	16	0	0.016	0
	10000	238	30	0.0238	0.0030
	100000	1899	482	0.019	0.0048
G=[13,15]	1000	41	3	0.0411	0.0032
	10000	252	23	0.0252	0.0023
	100000	2275	364	0.0228	0.0036
G=[133,171]	1000	38	0	0.0381	0
	10000	267	5	0.0267	5.0241e-004
	100000	3079	64	0.0308	6.4031e-004

Table 2: Comparing BER for 3 Coders

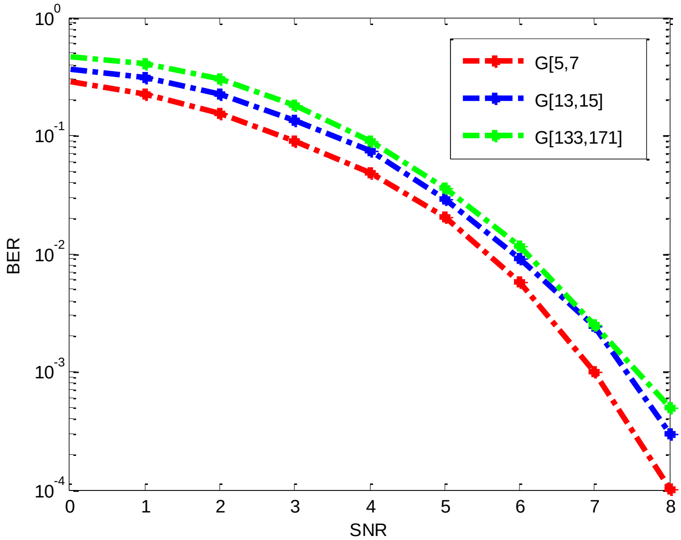

Figure 7: BER Vs SNR for Soft Decision Decoding

In Fig.7, for soft decision decoding, when the SNR value increases, BER drops. Lower constraint length coder give good performance for low SNR, but higher constraint length coder becomes better for higher SNR values.

For hard decision decoding also, it is observed from graph (Fig.4.8.) that the BER value decreases with SNR, for all the coders. When comparing the BER for 3 coders, G=[5,7] < G=[133,171] < G=[13,15] .Note the performance of G[13,15].

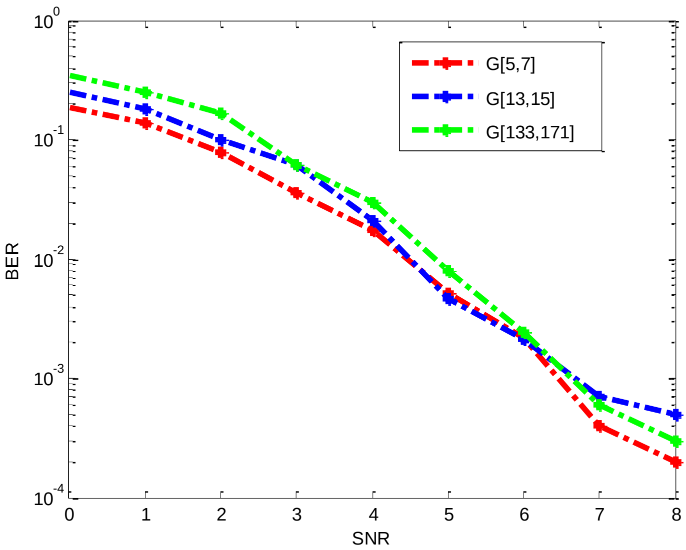

Figure 8: BER Vs SNR for Hard Decision Decoding

5 Conclusion

The demand of high transmission efficiency of communication systems is increasing every day. With the development of new technology data handling and storage become a big issue where coding plays a prominent role. The popularity of error correction codes led researchers to move deep into the existing coding theory and develop new methods to improve the performance. Trellises become an underlying theory to reduce the encoding and decoding complexity because of its great advantage of reduced computational processes. Error detection and path control become more vivid when it comes to a graphical analysis. The low bit error rate gives the choice of coders to be used, selected for a suitable length of data through a noisy environment.

The paper gives more importance to the trellis construction of different convolutional coders and the viterbi decoding steps. The performance of various coders is analyzed for different message lengths and for both hard and soft decision decoding. BER Vs SNR graphical comparison of three different coders gives the performance of the system at different noise levels. The future works includes a detailed study of trellis oriented generator matrix for different types of block codes, particularly for turbo codes, which is widely used in satellite and deep space communication. Minimal state trellises and state –space trellises are emerging techniques in this field.

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